

TECHNICAL NOTE

D-1013

THE MAGNETIC FIELD ON THE AXIS OF CIRCULAR CYLINDRICAL COILS

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CIRCULAR CYLINDRICAL COILS

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SUMMARY

The magnetic-field intensity along the axis of circular cylindrical coils of finite winding thickness has been computed for a range of coil geometries. Nondimensional field plots are presented that facilitate coil design for a desired field intensity along the axis.

INTRODUCTION

A research program at the Langley Research Center of the National Aeronautics and Space Administration required the design of aircore coils to generate magnetic fields of desired strength and distribution along the axis. Efficient design required a knowledge of the field intensity along the axis generated by coils of various lengths, diameters, and thicknesses.

The magnetic field intensity along the axis of the ideal solenoid, that is, a coil with an infinitesimally thin winding, is presented in reference 1 for a wide variety of coil geometries. (Ref. 1 presents the field intensity on and off the axis of the ideal solenoid.) The magnetic field intensity variation along the axis of a large number of thick-winding coil configurations is presented in reference 2. The range of coil geometries considered in reference 2, however, does not include a large part of the thick-winding coil geometry range of present interest.

In the present study computations of magnetic field intensity along the axis of coils of finite winding thickness were made for a range of coil geometries. The present study provides an extension to the range of coil geometries considered in reference 2. Nondimensional field plots are presented that facilitate coil design for a desired field intensity along the axis.

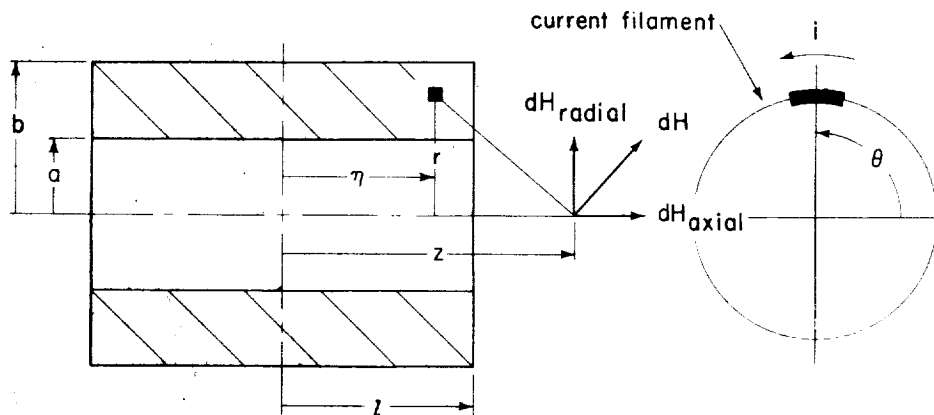
SYMBOLS

a	inside radius of coil
b	outside radius of coil
H	rationalized magnetic-field intensity
i	current in filament
J	current density
l	half length of coil
r	radial position of current filament
z	distance along axis from center of coil
η	axial position of current filament
θ	angular position along current filament

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METHOD OF COMPUTATION

Consider a circular cylindrical coil of arbitrary dimensions with the cylindrical coordinates z , r , and θ as shown in the following sketch:



The incremental component of H at a point on the axis in the axial direction due to an elemental volume of the filament current is

$$dH = \frac{1}{4\pi} \frac{r^2}{[r^2 + (z - \eta)^2]^{3/2}} d\theta \quad (1)$$

(Due to the symmetry of the coil, the incremental components of H in the radial direction cancel one another.)

By definition,

$$i = J d\eta dr \quad (2)$$

and

$$dH = \frac{J}{4\pi} \frac{r^2}{[r^2 + (z - \eta)^2]^{3/2}} d\theta d\eta dr \quad (3)$$

or

$$H = \frac{1}{4\pi} \int_a^b \int_{-l}^l \int_0^{2\pi} J \frac{r^2}{[r^2 + (z - \eta)^2]^{3/2}} d\theta d\eta dr \quad (4)$$

Evaluation of equation (4) for uniform J yields (see, for example, ref. 3)

$$H = \frac{J}{2} \left[(l + z) \log_e \frac{b + \sqrt{b^2 + (l + z)^2}}{a + \sqrt{a^2 + (l + z)^2}} + (l - z) \log_e \frac{b + \sqrt{b^2 + (l - z)^2}}{a + \sqrt{a^2 + (l - z)^2}} \right] \quad (5)$$

Magnetic-field intensity H was divided by current density J and the reference length l to give a nondimensional expression H/Jl .

RESULTS AND DISCUSSION

The variation of H/Jl with thickness-half length ratio $\frac{b-a}{l}$ is presented in figure 1 for various axial-position ratios z/l . Note that

increases of the ratio of inside radius to half length decrease the magnitude of the variation of $\frac{H}{Jl}$ along the axis for all values of $\frac{b-a}{l}$.

Cross plots of the variation of H/Jl with z/l (fig. 2) show a considerable decrease in H/Jl outside the coil ($\frac{z}{l} > 1.0$) for coils of small a/l . Coils of large a/l show gradual changes of H/Jl from the coil center ($\frac{z}{l} = 0$) to $\frac{z}{l} = 2$, particularly for thin windings. At the center of the coil ($\frac{z}{l} = 0$), the variation of H/Jl with z/l is zero, and the maximum variation occurs near the end of the coil ($\frac{z}{l} = 1$).

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Cross plots of the variation of H/Jl with a/l are presented in figure 3. At the coil center ($\frac{z}{l} = 0$) and end ($\frac{z}{l} = 1$), H/Jl decreases as a/l increases, whereas at a point one coil length from the center ($\frac{z}{l} = 2$) H/Jl increases as a/l increases.

Illustrations of the use of the variations of field intensity with coil geometry presented in this paper may be helpful. Two examples follow:

Example 1. - A coil is assumed to have the following dimensions: $l = 4$ cm, $a = 5$ cm, and $b = 13$ cm. Find the current density necessary to obtain a maximum field intensity of $0.9 \times 10^6 \frac{\text{amperes}}{\text{meter}}$ (about 11,300 oersteds) on the coil axis.

The maximum field intensity occurs at the coil center, $z/l = 0$. For $\frac{a}{l} = 1.25$ and $\frac{b-a}{l} = 2.0$, from figure 3(e)

$$\frac{H}{Jl} = 0.84$$

Therefore,

$$J = \frac{0.9 \times 10^6}{0.84(0.04)} = 26.7 \times 10^6 \frac{\text{amperes}}{\text{meter}^2}$$

Example 2. - A coil is assumed to have the following dimensions:
 $l = 10$ cm and $a = 5$ cm. For a maximum current density of
 $5 \times 10^6 \frac{\text{amperes}}{\text{meter}^2}$, find the minimum winding thickness necessary to obtain
 a field intensity of at least $0.4 \times 10^6 \frac{\text{amperes}}{\text{meter}}$ (about 5,000 oersteds)
 over an axial distance of 16 cm (8 cm from the center of the coil in
 both directions).

The minimum field intensity occurs at the maximum axial distance
 from the coil center, so that the axial point of evaluation must be at
 $z = 8$ cm or $z/l = 0.8$.

$$\frac{H}{Jl} = \frac{0.4 \times 10^6}{(5 \times 10^6)(0.1)} = 0.8$$

The value of

$$\frac{b - a}{l} = 1.63$$

is found in figure 1(b). Therefore,

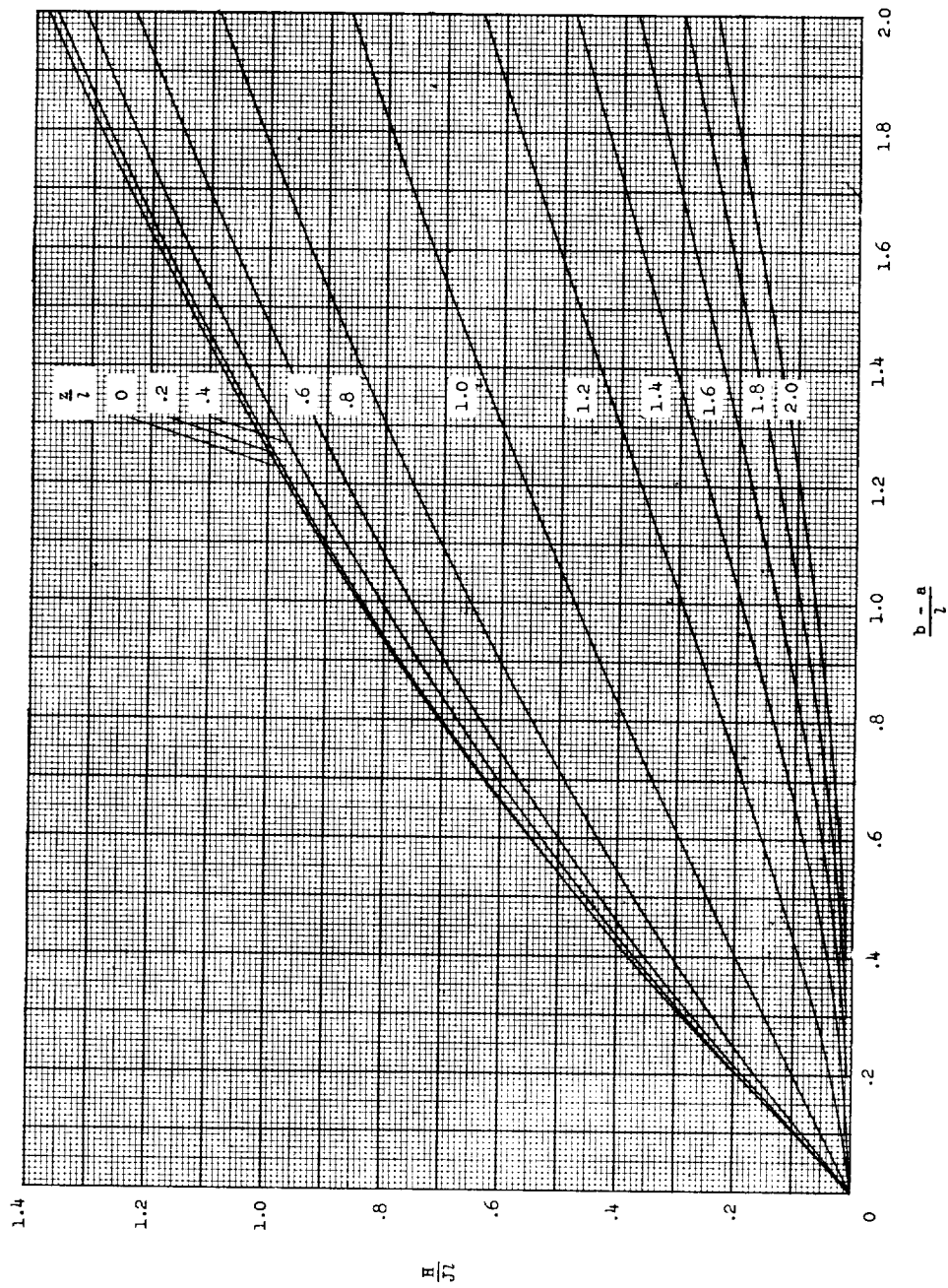
$$b - a = 16.3 \text{ cm}$$

Langley Research Center,
 National Aeronautics and Space Administration,
 Langley Air Force Base, Va., November 3, 1961.

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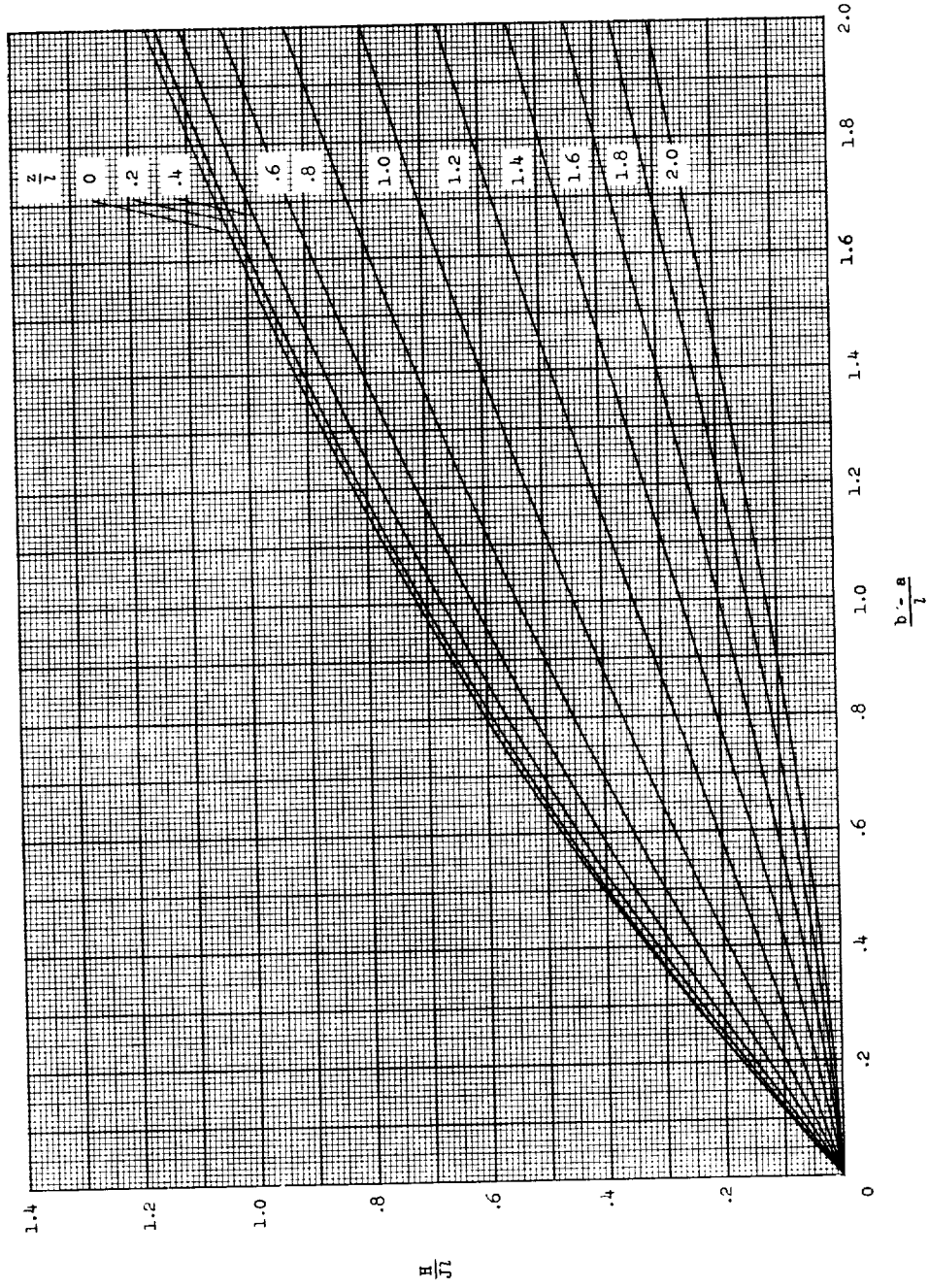
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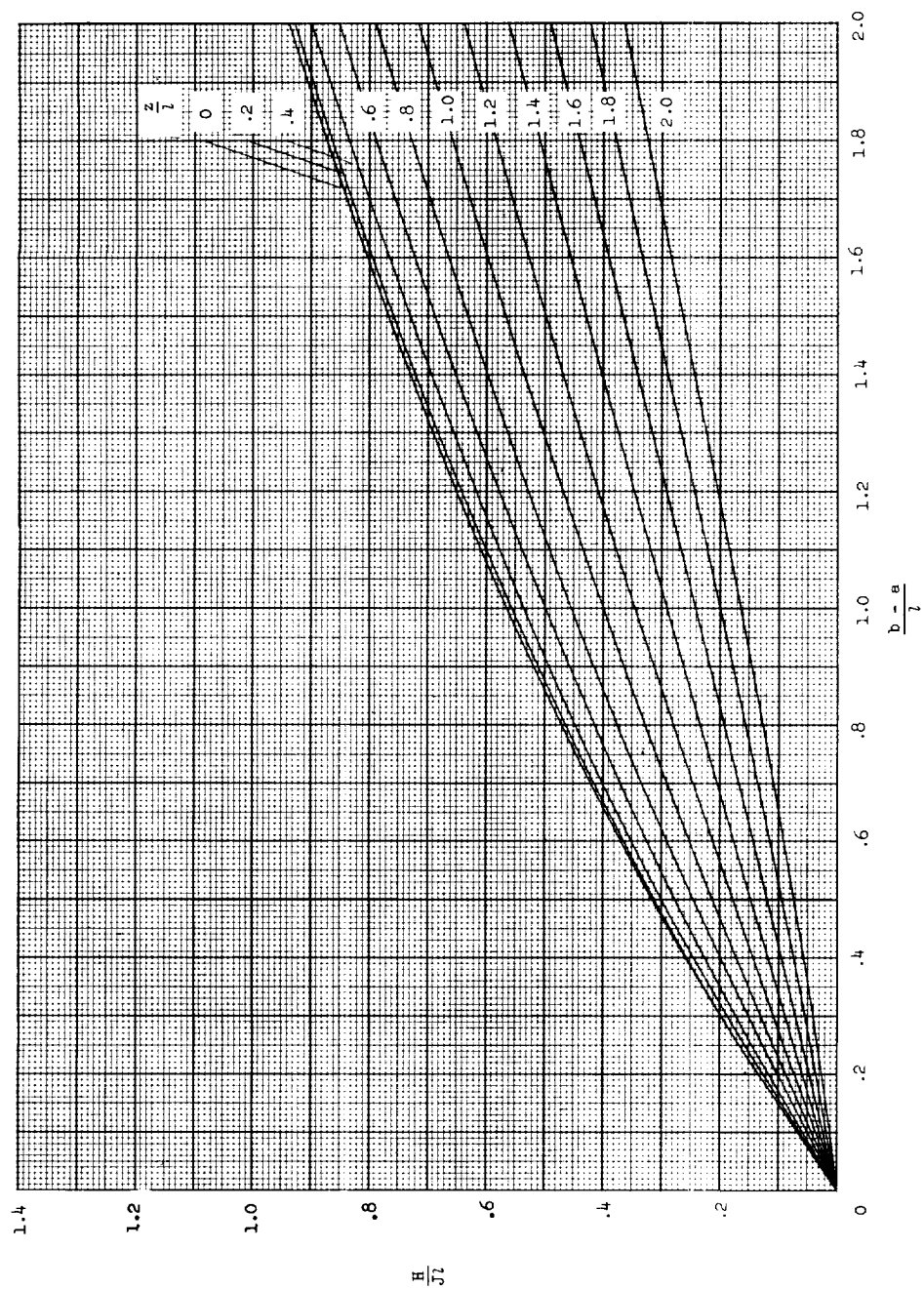
(a) $\frac{a}{l} = 0.125$.

Figure 1.- Variation of nondimensional magnetic-field intensity with winding thickness to half-length ratio.



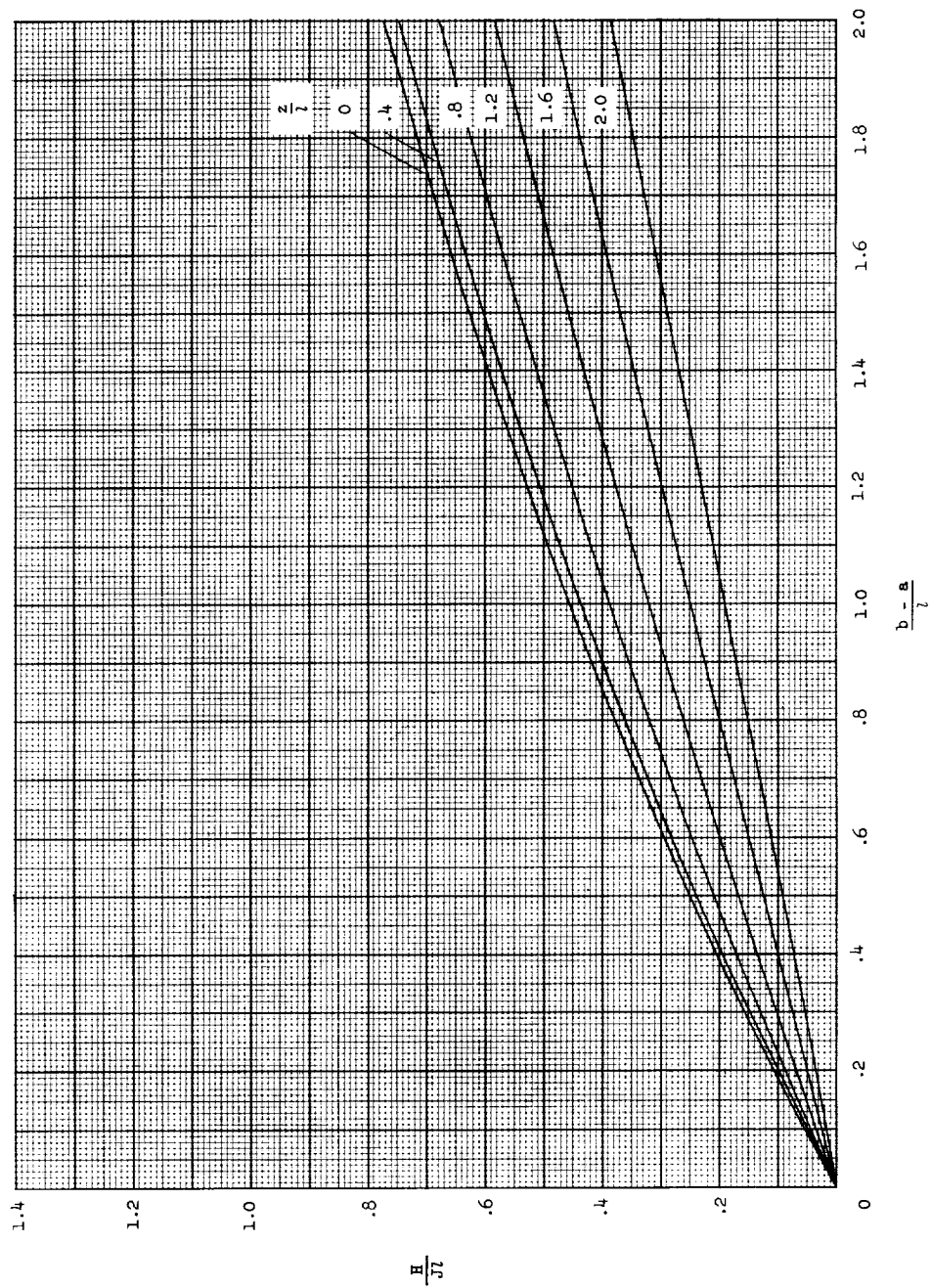
(b) $\frac{a}{l} = 0.500$.

Figure 1.- Continued.



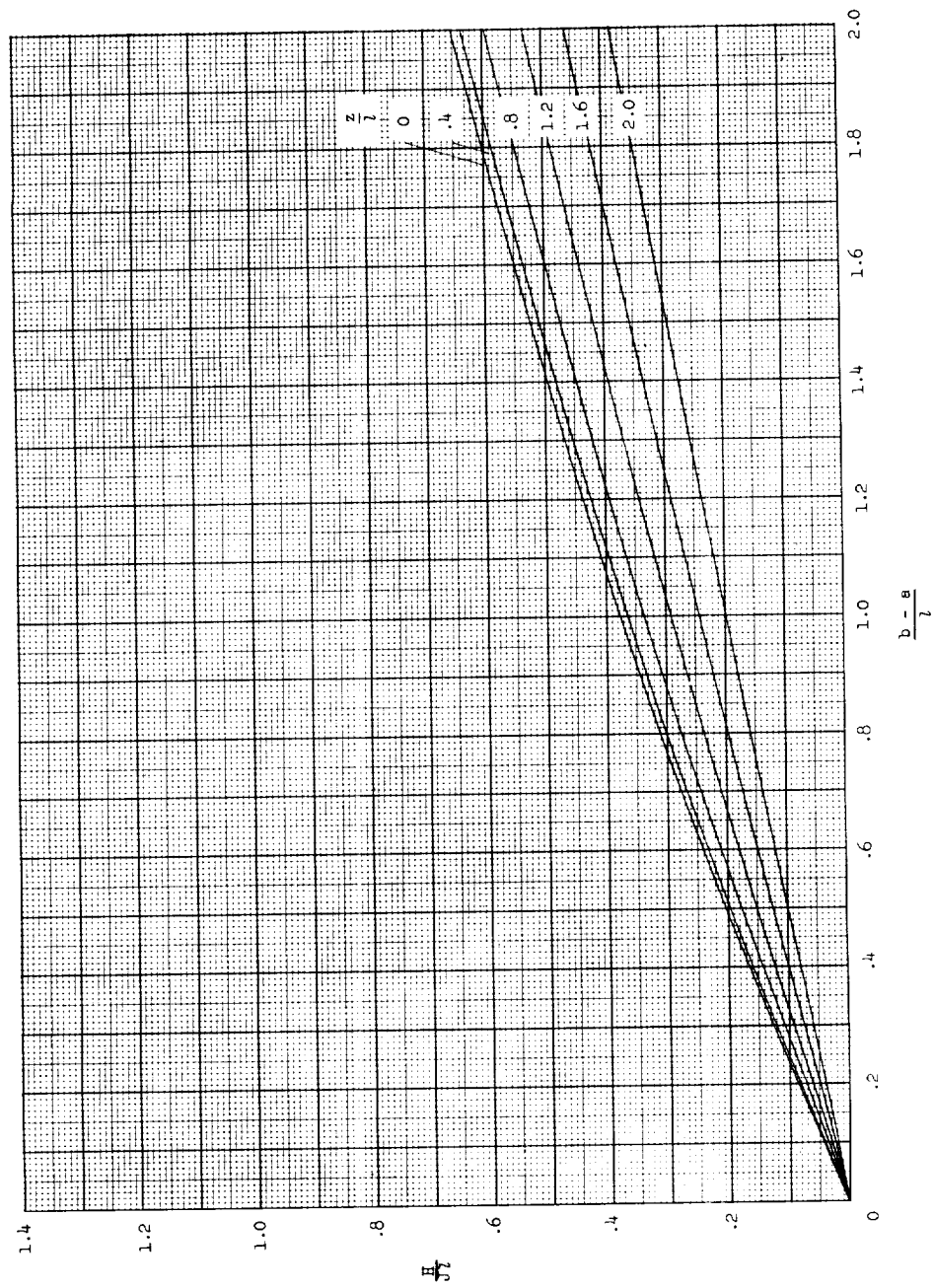
(c) $\frac{a}{l} = 1.000$.

Figure 1.- Continued.



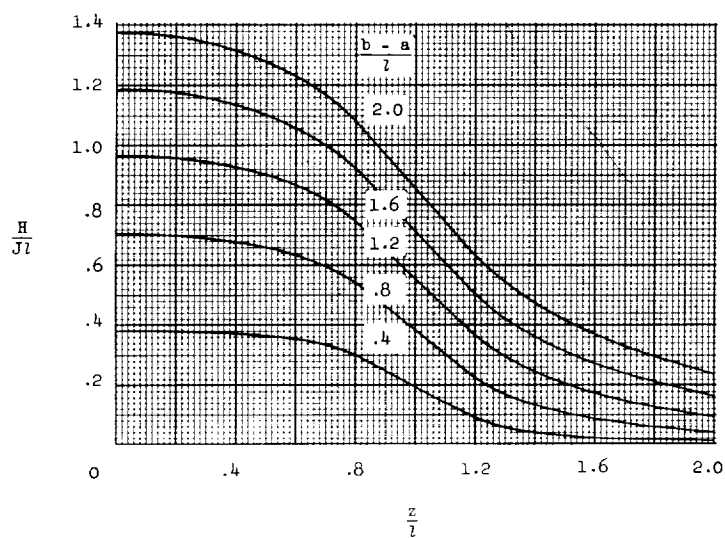
(d) $\frac{a}{l} = 1.500$.

Figure 1.- Continued.

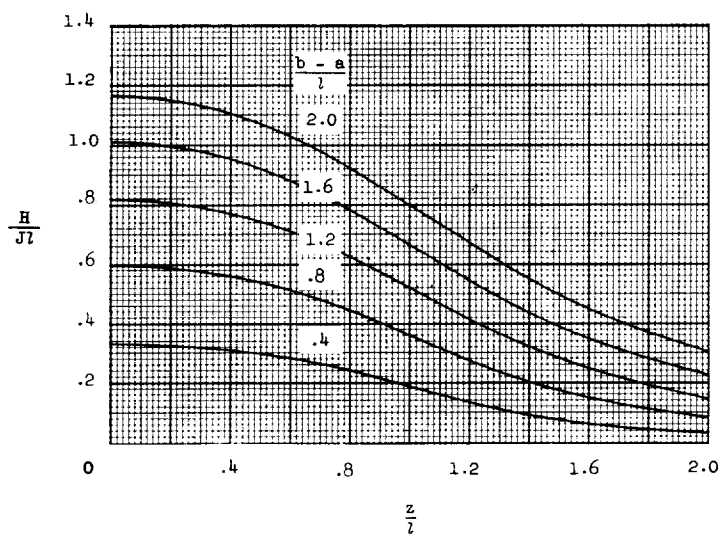


(e) $\frac{a}{l} = 2.000$.

Figure 1.- Concluded.



(a) $\frac{a}{l} = 0.125$.



(b) $\frac{a}{l} = 0.500$.

Figure 2.- Variation of nondimensional magnetic-field intensity with axial position.

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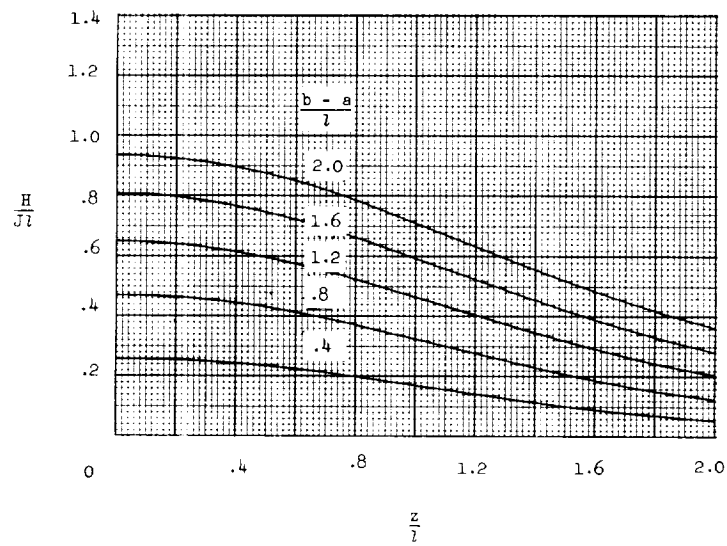
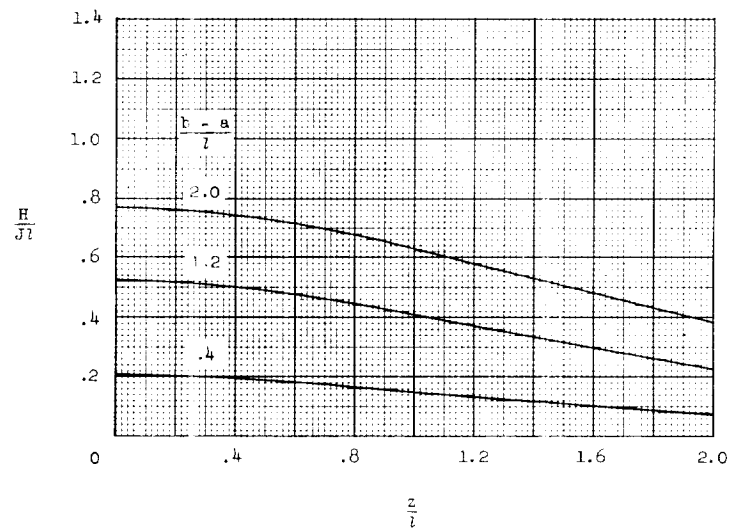
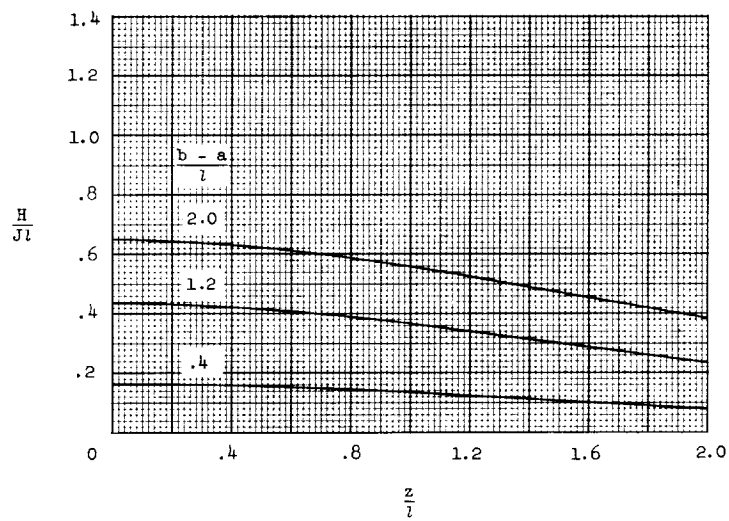
(c) $\frac{a}{l} = 1.000$.(d) $\frac{a}{l} = 1.500$.

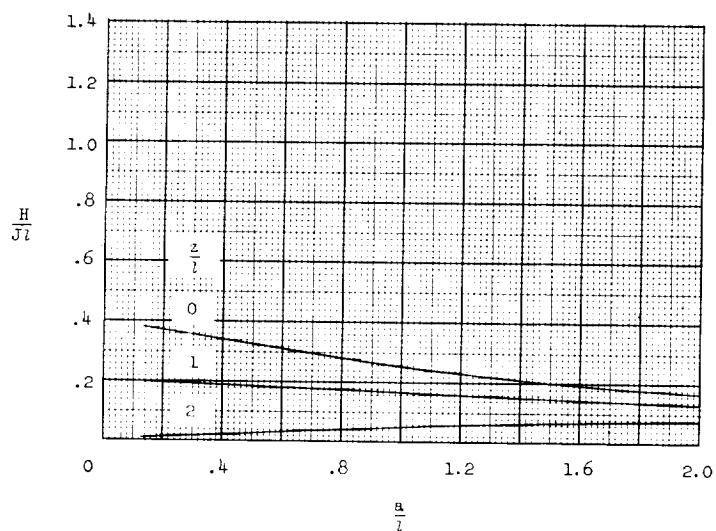
Figure 2.- Continued.



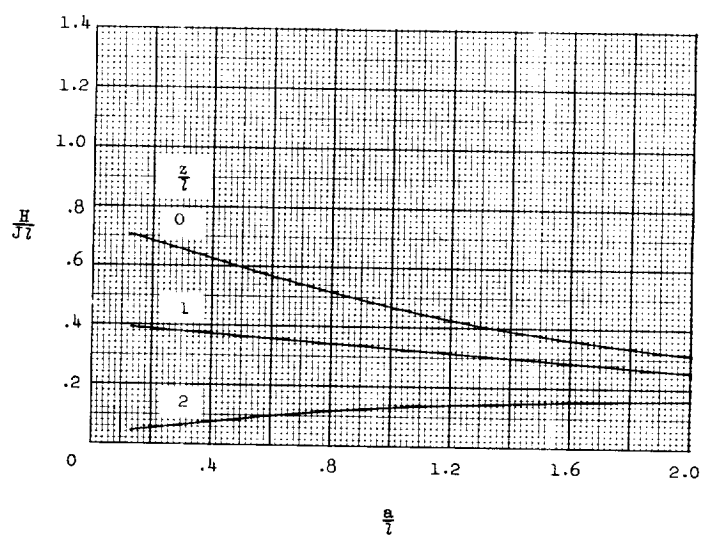
(e) $\frac{a}{l} = 2.000$.

Figure 2.- Concluded.

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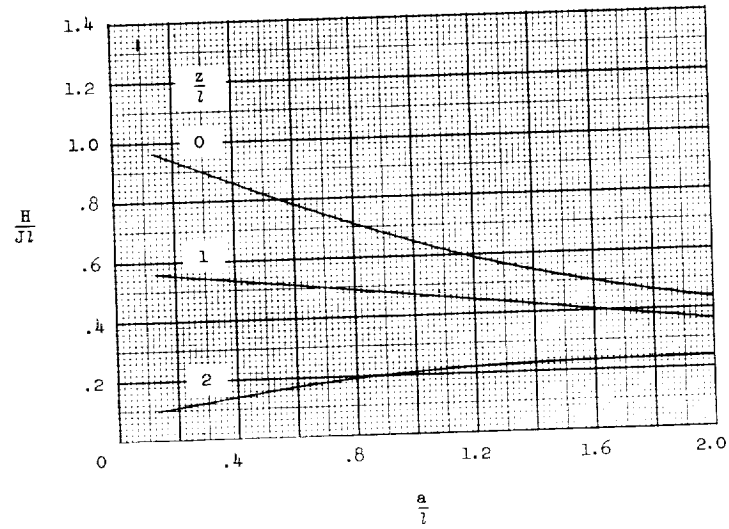


$$(a) \quad \frac{b-a}{l} = 0.4.$$

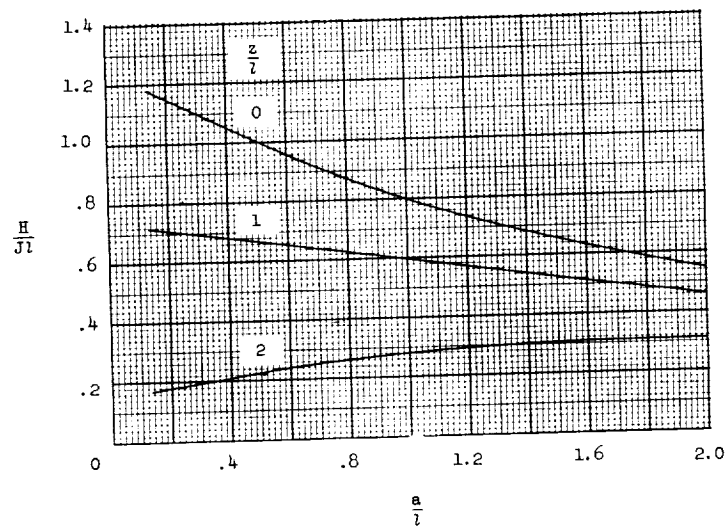


$$(b) \quad \frac{b-a}{l} = 0.8.$$

Figure 3.- Variation of nondimensional magnetic-field intensity with inside radius to half-length ratio.

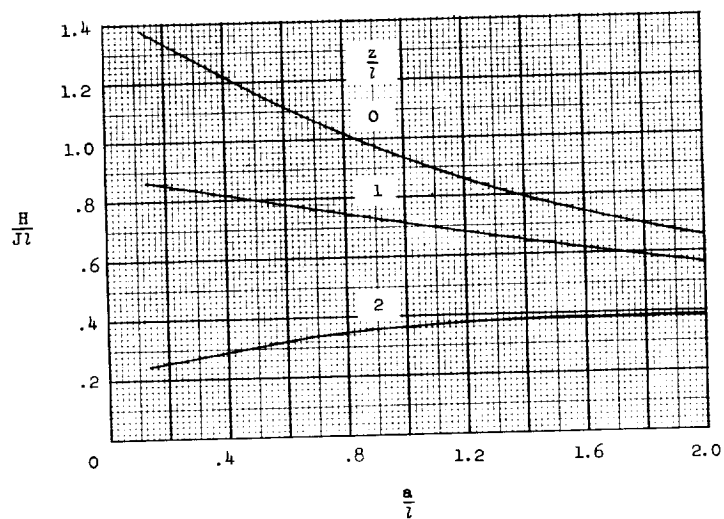


(c) $\frac{b-a}{l} = 1.2.$



(d) $\frac{b-a}{l} = 1.6.$

Figure 3.- Continued.



(e) $\frac{b-a}{l} = 2.0.$

Figure 3.- Concluded.